CK-12 Probability and Statistics - Advanced (Second Edition)

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Chapter 10

Chi-Square

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Printed: December 17, 2014

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CHAPTER 10 Chi-Square

Chapter Outline

10.1 The Goodness-of-Fit Test

Learning Objectives

- Understand the difference between the chi-square distribution and Student's t-distribution.
- Identify the conditions which must be satisfied when using the chi-square test.
- Understand the features of experiments that allow goodness-of-fit tests to be used.
- Evaluate a hypothesis using the goodness-of-fit test.

Introduction

In previous lessons, we learned that there are several different tests that we can use to analyze data and test hypotheses. The type of test that we choose depends on the data available and what question we are trying to answer. We analyze simple descriptive statistics, such as the mean, median, mode, and standard deviation to give us an idea of the distribution and to remove outliers, if necessary. We calculate probabilities to determine the likelihood of something happening. Finally, we use regression analysis to examine the relationship between two or more continuous variables.

However, there is another test that we have yet to cover. To analyze patterns between distinct categories, such as genders, political candidates, locations, or preferences, we use the chi-square test.

This test is used when estimating how closely a sample matches the expected distribution (also known as the goodness-of-fit test) and when estimating if two random variables are independent of one another (also known as the test of independence).

In this lesson, we will learn more about the goodness-of-fit test and how to create and evaluate hypotheses using this test.

The Chi-Square Distribution

The *chi-square distribution* can be used to perform the *goodness-of-fit test*, which compares the observed values of a categorical variable with the expected values of that same variable.

Example: We would use the chi-square goodness-of-fit test to evaluate if there was a preference in the type of lunch that 11^{th} grade students bought in the cafeteria. For this type of comparison, it helps to make a table to visualize the problem. We could construct the following table, known as a *contingency table*, to compare the observed and expected values.

Research Question: Do 11^{th} grade students prefer a certain type of lunch?

Using a sample of 11^{th} grade students, we recorded the following information:

TABLE 10.1: Frequency of Type of School Lunch Chosen by Students

Type of Lunch	Observed Frequency	Expected Frequency
Salad	21	25

TABLE 10.1: (continued)

Type of Lunch	Observed Frequency	Expected Frequency
Sub Sandwich	29	25
Daily Special	14	25
Brought Own Lunch	36	25

If there is no difference in which type of lunch is preferred, we would expect the students to prefer each type of lunch equally. To calculate the expected frequency of each category when assuming school lunch preferences are distributed equally, we divide the number of observations by the number of categories. Since there are 100 observations and 4 categories, the expected frequency of each category is $\frac{100}{4}$, or 25.

The value that indicates the comparison between the observed and expected frequency is called the *chi-square statistic*. The idea is that if the observed frequency is close to the expected frequency, then the chi-square statistic will be small. On the other hand, if there is a substantial difference between the two frequencies, then we would expect the chi-square statistic to be large.

To calculate the chi-square statistic, χ^2, we use the following formula:

$\chi^2 = \sum \frac{(O-E)^2}{E}$

where:

χ^2 is the chi-square test statistic.

O is the observed frequency value for each event.

E is the expected frequency value for each event.

We compare the value of the test statistic to a tabled chi-square value to determine the probability that a sample fits an expected pattern.

Features of the Goodness-of-Fit Test

As mentioned, the goodness-of-fit test is used to determine patterns of distinct categorical variables. The test requires that the data are obtained through a random sample. The number of *degrees of freedom* associated with a particular chi-square test is equal to the number of categories minus one. That is, $df = c - 1$.

Example: Using our example about the preferences for types of school lunches, we calculate the degrees of freedom as follows:

$$\begin{aligned} df &= \text{number of categories} - 1 \\ 3 &= 4 - 1 \end{aligned}$$

There are many situations that use the goodness-of-fit test, including surveys, taste tests, and analysis of behaviors. Interestingly, goodness-of-fit tests are also used in casinos to determine if there is cheating in games of chance, such as cards or dice. For example, if a certain card or number on a die shows up more than expected (a high observed frequency compared to the expected frequency), officials use the goodness-of-fit test to determine the likelihood that the player may be cheating or that the game may not be fair.

Evaluating Hypotheses Using the Goodness-of-Fit Test

Let's use our original example to create and test a hypothesis using the goodness-of-fit chi-square test. First, we will need to state the null and alternative hypotheses for our research question. Since our research question asks, "Do

11^{th} grade students prefer a certain type of lunch?" our null hypothesis for the chi-square test would state that there is no difference between the observed and the expected frequencies. Therefore, our alternative hypothesis would state that there is a significant difference between the observed and expected frequencies.

Null Hypothesis

$H_0 : O = E$ (There is no statistically significant difference between observed and expected frequencies.)

Alternative Hypothesis

$H_a : O \neq E$ (There is a statistically significant difference between observed and expected frequencies.)

Also, the number of degrees of freedom for this test is 3.

Using an alpha level of 0.05, we look under the column for 0.05 and the row for degrees of freedom, which, again, is 3. According to the standard chi-square distribution table, we see that the critical value for chi-square is 7.815. Therefore, we would reject the null hypothesis if the chi-square statistic is greater than 7.815.

Note that we can calculate the chi-square statistic with relative ease.

TABLE 10.2: Frequency Which Student Select Type of School Lunch

Type of Lunch	Observed Frequency	Expected Frequency	$\frac{(O-E)^2}{E}$
Salad	21	25	0.64
Sub Sandwich	29	25	0.64
Daily Special	14	25	4.84
Brought Own Lunch	36	25	4.84
Total (chi-square)			10.96

Since our chi-square statistic of 10.96 is greater than 7.815, we reject the null hypotheses and accept the alternative hypothesis. Therefore, we can conclude that there is a significant difference between the types of lunches that 11^{th} grade students prefer.

Lesson Summary

We use the chi-square test to examine patterns between categorical variables, such as genders, political candidates, locations, or preferences.

There are two types of chi-square tests: the goodness-of-fit test and the test for independence. We use the goodness-of-fit test to estimate how closely a sample matches the expected distribution.

To test for significance, it helps to make a table detailing the observed and expected frequencies of the data sample. Using the standard chi-square distribution table, we are able to create criteria for accepting the null or alternative hypotheses for our research questions.

To test the null hypothesis, it is necessary to calculate the chi-square statistic, χ^2. To calculate the chi-square statistic, we use the following formula:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where:

χ^2 is the chi-square test statistic.

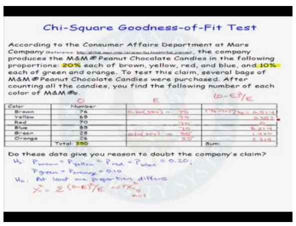
O is the observed frequency value for each event.

E is the expected frequency value for each event.

Using the chi-square statistic and the level of significance, we are able to determine whether to reject or fail to reject the null hypothesis and write a summary statement based on these results.

Multimedia Links

For a discussion on P-value and an example of a chi-square goodness of fit test **(7.0)(14.0)(18.0)(19.0)**, see APUS 07, Example of a Chi-Square Goodness-of-Fit Test (8:45).



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URL: http://www.ck12.org/flx/render/embeddedobject/1055

Follow this link to a table of chi-square values: http://tinyurl.com/3ypvj2h

Review Questions

1. What is the name of the statistical test used to analyze the patterns between two categorical variables?
   a. Student's t-test
   b. the ANOVA test
   c. the chi-square test
   d. the z-score
2. There are two types of chi-square tests. Which type of chi-square test estimates how closely a sample matches an expected distribution?
   a. the goodness-of-fit test
   b. the test for independence
3. Which of the following is considered a categorical variable?
   a. income
   b. gender
   c. height
   d. weight
4. If there were 250 observations in a data set and 2 uniformly distributed categories that were being measured, the expected frequency for each category would be:
   a. 125
   b. 500
   c. 250
   d. 5
5. What is the formula for calculating the chi-square statistic?

6. A principal is planning a field trip. She samples a group of 100 students to see if they prefer a sporting event, a play at the local college, or a science museum. She records the following results:

TABLE 10.3:

Type of Field Trip	Number Preferring
Sporting Event	53
Play	18
Science Museum	29

(a) What is the observed frequency value for the Science Museum category?

(b) What is the expected frequency value for the Sporting Event category?

(c) What would be the null hypothesis for the situation above?

(i) There is no preference between the types of field trips that students prefer.

(ii) There is a preference between the types of field trips that students prefer.

(d) What would be the chi-square statistic for the research question above?

(e) If the estimated chi-square level of significance was 5.99, would you reject or fail to reject the null hypothesis?

On the Web

http://onlinestatbook.com/stat_sim/chisq_theor/index.html Explore what happens when you are using the chi-square statistic when the underlying population from which you are sampling does not follow a normal distribution.

10.2 Test of Independence

Learning Objectives

- Understand how to draw data needed to perform calculations when running the chi-square test from contingency tables.
- Run the test of independence to determine whether two variables are independent or not.
- Use the test of homogeneity to examine the proportions of a variable attributed to different populations.

Introduction

As mentioned in the previous lesson, the chi-square test can be used to both estimate how closely an observed distribution matches an expected distribution (the goodness-of-fit test) and to estimate whether two random variables are independent of one another (the test of independence). In this lesson, we will examine the test of independence in greater detail.

The chi-square test of independence is used to assess if two factors are related. This test is often used in social science research to determine if factors are independent of each other. For example, we would use this test to determine relationships between voting patterns and race, income and gender, and behavior and education.

In general, when running the test of independence, we ask, "Is Variable X independent of Variable Y?" It is important to note that this test does not test how the variables are related, just simply whether or not they are independent of one another. For example, while the test of independence can help us determine if income and gender are independent, it cannot help us assess how one category might affect the other.

Drawing Data from Contingency Tables Needed to Perform Calculations when Running a Chi-Square Test

Contingency tables can help us frame our hypotheses and solve problems. Often, we use contingency tables to list the variables and observational patterns that will help us to run a chi-square test. For example, we could use a contingency table to record the answers to phone surveys or observed behavioral patterns.

Example: We would use a contingency table to record the data when analyzing whether women are more likely to vote for a Republican or Democratic candidate when compared to men. In this example, we want to know if voting patterns are independent of gender. Hypothetical data for 76 females and 62 males from the state of California are in the contingency table below.

TABLE 10.4: Frequency of California Citizens voting for a Republican or Democratic Candidate

	Democratic	**Republican**	**Total**
Female	48	28	76
Male	36	26	62
Total	84	54	138

Similar to the chi-square goodness-of-fit test, the *test of independence* is a comparison of the differences between observed and expected values. However, in this test, we need to calculate the expected value using the row and column totals from the table. The expected value for each of the potential outcomes in the table can be calculated using the following formula:

$$\text{Expected Frequency} = \frac{(\text{Row Total})(\text{Column Total})}{\text{Total Number of Observations}}$$

In the table above, we calculated the row totals to be 76 females and 62 males, while the column totals are 84 Democrats and 54 Republicans. Using the formula, we find the following expected frequencies for the potential outcomes:

The expected frequency for female Democratic outcome is $76 \bullet \frac{84}{138} = 46.26$.

The expected frequency for female Republican outcome is $76 \bullet \frac{54}{138} = 29.74$.

The expected frequency for male Democratic outcome is $62 \bullet \frac{84}{138} = 37.74$.

The expected frequency for male Republican outcome is $62 \bullet \frac{54}{138} = 24.26$.

Using these calculated expected frequencies, we can modify the table above to look something like this:

TABLE 10.5:

	Democratic Observed	**Democratic** Expected	**Republican** Observed	**Republican** Expected	**Total**
Female	48	46.26	28	29.74	76
Male	36	37.74	26	24.26	62
Total	84		54		138

With the figures above, we are able to calculate the chi-square statistic with relative ease.

The Chi-Square Test of Independence

When running the test of independence, we use similar steps as when running the goodness-of-fit test described earlier. First, we need to establish a hypothesis based on our research question. Using our scenario of gender and voting patterns, our null hypothesis is that there is not a significant difference in the frequencies with which females vote for a Republican or Democratic candidate when compared to males. Therefore, our hypotheses can be stated as follows:

Null Hypothesis

$H_0 : O = E$ (There is no statistically significant difference between the observed and expected frequencies.)

Alternative Hypothesis

$H_a : O \neq E$ (There is a statistically significant difference between the observed and expected frequencies.)

Using the table above, we can calculate the degrees of freedom and the chi-square statistic. The formula for calculating the chi-square statistic is the same as before:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where:

χ^2 is the chi-square test statistic.

O is the observed frequency value for each event.

E is the expected frequency value for each event.

Using this formula and the example above, we get the following expected frequencies and chi-square statistic:

TABLE 10.6:

	Democratic Obs. Freq.	Democratic Exp. Freq.	Democratic $\frac{(O-E)^2}{E}$	Republican Obs. Freq.	Republican Exp. Freq.	Republican $\frac{(O-E)^2}{E}$
Female	48	46.26	0.07	28	29.74	0.10
Male	36	37.74	0.08	26	24.26	0.12
Totals	84			54		

$$\chi^2 = 0.07 + 0.08 + 0.10 + 0.12 = 0.37$$

Also, the degrees of freedom can be calculated from the number of Columns ("C") and the number of Rows ("R") as follows:

$$\begin{aligned} df &= (C-1)(R-1) \\ &= (2-1)(2-1) = 1 \end{aligned}$$

With an alpha level of 0.05, we look under the column for 0.05 and the row for degrees of freedom, which, again, is 1, in the standard chi-square distribution table (http://tinyurl.com/3ypvj2h). According to the table, we see that the critical value for chi-square is 3.841. Therefore, we would reject the null hypothesis if the chi-square statistic is greater than 3.841.

Since our calculated chi-square value of 0.37 is less than 3.841, we fail to reject the null hypothesis. Therefore, we can conclude that females are not significantly more likely to vote for a Republican or Democratic candidate than males. In other words, these two factors appear to be independent of one another.

On the Web

http://tinyurl.com/39lhc3y A chi-square applet demonstrating the test of independence.

Test of Homogeneity

The chi-square goodness-of-fit test and the test of independence are two ways to examine the relationships between categorical variables. To determine whether or not the assignment of categorical variables is random (that is, to examine the randomness of a sample), we perform the *test of homogeneity*. In other words, the test of homogeneity tests whether samples from populations have the same proportion of observations with a common characteristic. For example, we found in our last test of independence that the factors of gender and voting patterns were independent of one another. However, our original question was if females were more likely to vote for a Republican or Democratic candidate when compared to males. We would use the test of homogeneity to examine the probability that choosing a Republican or Democratic candidate was the same for females and males.

Another commonly used example of the test of homogeneity is comparing dice to see if they all work the same way.

Example: The manager of a casino has two potentially loaded dice that he wants to examine. (Loaded dice are ones that are weighted on one side so that certain numbers have greater probabilities of showing up.) The manager rolls each of the dice exactly 20 times and comes up with the following results:

TABLE 10.7: Number Rolled with the Potentially Loaded Dice

	1	**2**	**3**	**4**	**5**	**6**	**Totals**
Die 1	6	1	2	2	3	6	20
Die 2	4	1	3	3	1	8	20
Totals	10	2	5	5	4	14	40

Like the other chi-square tests, we first need to establish a null hypothesis based on a research question. In this case, our research question would be something like, "Is the probability of rolling a specific number the same for Die 1 and Die 2?" This would give us the following hypotheses:

Null Hypothesis

$H_0 : O = E$ (The probabilities are the same for both dice.)

Alternative Hypothesis

$H_a : O \neq E$ (The probabilities differ for both dice.)

Similar to the test of independence, we need to calculate the expected frequency for each potential outcome and the total number of degrees of freedom. To get the expected frequency for each potential outcome, we use the same formula as we used for the test of independence, which is as follows:

$$\text{Expected Frequency} = \frac{(\text{Row Total})(\text{Column Total})}{\text{Total Number of Observations}}$$

The following table includes the expected frequency (in parenthesis) for each outcome, along with the chi-square statistic, $\chi^2 = \frac{(O-E)^2}{E}$, in a separate column:

Number Rolled on the Potentially Loaded Dice

TABLE 10.8:

	1	χ^2	**2**	χ^2	**3**	χ^2	**4**	χ^2	**5**	χ^2	**6**	χ^2	χ^2 **Total**
Die 1	6(5)	0.2	1(1)	0	2(2.5)	0.1	2(2.5)	0.1	3(2)	0.5	6(7)	0.14	1.04
Die 2	4(5)	0.2	1(1)	0	3(2.5)	0.1	3(2.5)	0.1	1(2)	0.5	8(7)	0.14	1.04
Totals	10		2		5		5		4		14		2.08

$$\begin{aligned} df &= (C-1)(R-1) \\ &= (6-1)(2-1) = 5 \end{aligned}$$

From the table above, we can see that the value of the test statistic is 2.08.

Using an alpha level of 0.05, we look under the column for 0.05 and the row for degrees of freedom, which, again, is 5, in the standard chi-square distribution table. According to the table, we see that the critical value for chi-square is 11.070. Therefore, we would reject the null hypothesis if the chi-square statistic is greater than 11.070.

Since our calculated chi-square value of 2.08 is less than 11.070, we fail to reject the null hypothesis. Therefore, we can conclude that each number is just as likely to be rolled on one die as on the other. This means that if the dice are loaded, they are probably loaded in the same way or were made by the same manufacturer.

Lesson Summary

The chi-square test of independence is used to assess if two factors are related. It is commonly used in social science research to examine behaviors, preferences, measurements, etc.

As with the chi-square goodness-of-fit test, contingency tables help capture and display relevant information. For each of the possible outcomes in the table constructed to run a chi-square test, we need to calculate the expected frequency. The formula used for this calculation is as follows:

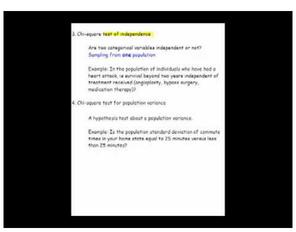
$$\text{Expected Frequency} = \frac{(\text{Row Total})(\text{Column Total})}{\text{Total Number of Observations}}$$

To calculate the chi-square statistic for the test of independence, we use the same formula as for the goodness-of-fit test. If the calculated chi-square value is greater than the critical value, we reject the null hypothesis.

We perform the test of homogeneity to examine the randomness of a sample. The test of homogeneity tests whether various populations are homogeneous or equal with respect to certain characteristics.

Multimedia Links

For a discussion of the four different scenarios for use of the chi-square test **(19.0)**, see American Public University, Test Requiring the Chi-Square Distribution (4:13).



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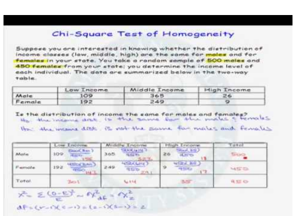
For an example of a chi-square test for homogenity **(19.0)**, see APUS07, Example of a Chi-Square Test of Homogenity (7:57).



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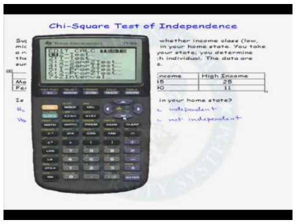
For an example of a chi-square test for independence with the TI-83/84 Calculator **(19.0)**, see APUS07, Example of a Chi-Square Test of Independence Using a Calculator (3:29).



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Review Questions

1. What is the chi-square test of independence used for?
2. True or False: In the test of independence, you can test if two variables are related, but you cannot test the nature of the relationship itself.
3. When calculating the expected frequency for a possible outcome in a contingency table, you use the formula:
   a. Expected Frequency $= \frac{(\text{Row Total})(\text{Column Total})}{\text{Total Number of Observations}}$
   b. Expected Frequency $= \frac{(\text{Total Observations})(\text{Column Total})}{\text{Row Total}}$
   c. Expected Frequency $= \frac{(\text{Total Observations})(\text{Row Total})}{\text{Column Total}}$
4. Use the table below to answer the following review questions.

TABLE 10.9: Research Question: Are females at UC Berkeley more likely to study abroad than males?

	Studied Abroad	**Did Not Study Abroad**
Females	322	460
Males	128	152

(a) What is the total number of females in the sample?

450

280

612

782

(b) What is the total number of observations in the sample?

782

533

1,062

612

(c) What is the expected frequency for the number of males who did not study abroad?

161

208

111

129

(d) How many degrees of freedom are in this example?

1

2

3

4

(e) True or False: Our null hypothesis would be that females are as likely as males to study abroad.

(f) What is the chi-square statistic for this example?

1.60

2.45

3.32

3.98

5. If the chi-square critical value at 0.05 and 1 degree of freedom is 3.81, and we have a calculated chi-square statistic of 2.22, we would:
   a. reject the null hypothesis
   b. fail to reject the null hypothesis
6. True or False: We use the test of homogeneity to evaluate the equality of several samples of certain variables.
7. The test of homogeneity is carried out the exact same way as:
   a. the goodness-of-fit test
   b. the test of independence

10.3 Testing One Variance

Learning Objectives

- Test a hypothesis about a single variance using the chi-square distribution.
- Calculate a confidence interval for a population variance based on a sample standard deviation.

Introduction

In the previous lesson, we learned how the chi-square test can help us assess the relationships between two variables. In addition to assessing these relationships, the chi-square test can also help us test hypotheses surrounding variance, which is the measure of the variation, or scattering, of scores in a distribution. There are several different tests that we can use to assess the variance of a sample. The most common tests used to assess variance are the chi-square test for one variance, the F-test, and the Analysis of Variance (ANOVA). Both the chi-square test and the F-test are extremely sensitive to non-normality (or when the populations do not have a normal distribution), so the ANOVA test is used most often for this analysis. However, in this section, we will examine in greater detail the testing of a single variance using the chi-square test.

Testing a Single Variance Hypothesis Using the Chi-Square Test

Suppose that we want to test two samples to determine if they belong to the same population. The test of variance between samples is used quite frequently in the manufacturing of food, parts, and medications, since it is necessary for individual products of each of these types to be very similar in size and chemical make-up. This test is called the *test for one variance*.

To perform the test for one variance using the chi-square distribution, we need several pieces of information. First, as mentioned, we should check to make sure that the population has a normal distribution. Next, we need to determine the number of observations in the sample. The remaining pieces of information that we need are the standard deviation and the hypothetical population variance. For the purposes of this exercise, we will assume that we will be provided with the standard deviation and the population variance.

Using these key pieces of information, we use the following formula to calculate the chi-square value to test a hypothesis surrounding single variance:

$$\chi^2 = \frac{df(s^2)}{\sigma^2}$$

where:

χ^2 is the chi-square statistical value.

$df = n - 1$, where n is the size of the sample.

s^2 is the sample variance.

σ^2 is the population variance.

We want to test the hypothesis that the sample comes from a population with a variance greater than the observed variance. Let's take a look at an example to help clarify.

Example: Suppose we have a sample of 41 female gymnasts from Mission High School. We want to know if their heights are truly a random sample of the general high school population with respect to variance. We know from a previous study that the standard deviation of the heights of high school women is 2.2.

To test this question, we first need to generate null and alternative hypotheses. Our null hypothesis states that the sample comes from a population that has a variance of less than or equal to 4.84 (σ^2 is the square of the standard deviation).

Null Hypothesis

$H_0 : \sigma^2 \leq 4.84$ (The variance of the female gymnasts is less than or equal to that of the general female high school population.)

Alternative Hypothesis

$H_a : \sigma^2 > 4.84$ (The variance of the female gymnasts is greater than that of the general female high school population.)

Using the sample of the 41 gymnasts, we compute the standard deviation and find it to be $s = 1.2$. Using the information from above, we calculate our chi-square value and find the following:

$$\chi^2 = \frac{(40)(1.2^2)}{4.84} = 11.9$$

Therefore, since 11.9 is less than 55.758 (the value from the chi-square table given an alpha level of 0.05 and 40 degrees of freedom), we fail to reject the null hypothesis and, therefore, cannot conclude that the female gymnasts have a significantly higher variance in height than the general female high school population.

Calculating a Confidence Interval for a Population Variance

Once we know how to test a hypothesis about a single variance, calculating a confidence interval for a population variance is relatively easy. Again, it is important to remember that this test is dependent on the normality of the population. For non-normal populations, it is best to use the *ANOVA test*, which we will cover in greater detail in another lesson. To construct a confidence interval for the population variance, we need three pieces of information: the number of observations in the sample, the variance of the sample, and the desired confidence interval. With the desired confidence interval, α (most often this is set at 0.10 to reflect a 90% confidence interval or at 0.05 to reflect a 95% confidence interval), we can construct the upper and lower limits around the significance level.

Example: We randomly select 30 containers of Coca Cola and measure the amount of sugar in each container. Using the formula that we learned earlier, we calculate the variance of the sample to be 5.20. Find a 90% confidence interval for the true variance. In other words, assuming that the sample comes from a normal population, what is the range of the population variance?

To construct this 90% confidence interval, we first need to determine our upper and lower limits. The formula to construct this confidence interval and calculate the population variance, σ^2, is as follows:

$$\frac{dfs^2}{\chi^2_{0.05}} \leq \sigma^2 \leq \frac{dfs^2}{\chi^2_{0.95}}$$

Using our standard chi-square distribution table (http://tinyurl.com/3ypvj2h), we can look up the critical χ^2 values for 0.05 and 0.95 at 29 degrees of freedom. According to the χ^2 distribution table, we find that $\chi^2_{0.05} = 42.557$ and that $\chi^2_{0.95} = 17.708$. Since we know the number of observations and the standard deviation for this sample, we can then solve for σ^2 as shown below:

$$\frac{dfs^2}{42.557} \le \sigma^2 \le \frac{dfs^2}{17.708}$$
$$\frac{150.80}{42.557} \le \sigma^2 \le \frac{150.80}{17.708}$$
$$3.54 \le \sigma^2 \le 8.52$$

In other words, we are 90% confident that the variance of the population from which this sample was taken is between 3.54 and 8.52.

Lesson Summary

We can also use the chi-square distribution to test hypotheses about population variance. Variance is the measure of the variation, or scattering, of scores in a distribution, and we often use this test to assess the likelihood that a population variance is within a certain range.

To perform the test for one variance using the chi-square statistic, we use the following formula:

$$\chi^2 = \frac{df(s^2)}{\sigma^2}$$

where:

χ^2 is the Chi-Square statistical value.

$df = n - 1$, where n is the size of the sample.

s^2 is the sample variance.

σ^2 is the population variance.

This formula gives us a chi-square statistic, which we can compare to values taken from the chi-square distribution table to test our hypothesis.

We can also construct a confidence interval, which is a range of values that includes the population variance with a given level of confidence. To find this interval, we use the formula shown below:

$$\frac{dfs^2}{\chi^2_{\frac{\alpha}{2}}} \le \sigma^2 \le \frac{dfs^2}{\chi^2_{1-\frac{\alpha}{2}}}$$

Review Questions

1. We use the chi-square distribution for the:
    a. goodness-of-fit test
    b. test for independence
    c. testing of a hypothesis of single variance

    d. all of the above

2. True or False: We can test a hypothesis about a single variance using the chi-square distribution for a non-normal population.
3. In testing variance around the population mean, our null hypothesis states that the two population means that we are testing are:
    a. equal with respect to variance
    b. not equal
    c. none of the above
4. In the formula for calculating the chi-square statistic for single variance, σ^2 is:
    a. standard deviation
    b. number of observations
    c. hypothesized population variance
    d. chi-square statistic
5. If we knew the number of observations in a sample, the standard deviation of the sample, and the hypothesized variance of the population, what additional information would we need to solve for the chi-square statistic?
    a. the chi-square distribution table
    b. the population size
    c. the standard deviation of the population
    d. no additional information is needed
6. We want to test a hypothesis about a single variance using the chi-square distribution. We weighed 30 bars of Dial soap, and this sample had a standard deviation of 1.1. We want to test if this sample comes from the general factory, which we know from a previous study to have an overall variance of 3.22. What is our null hypothesis?
7. Compute χ^2 for Question 6.
8. Given the information in Questions 6 and 7, would you reject or fail to reject the null hypothesis?
9. Let's assume that our population variance for this problem is unknown. We want to construct a 90% confidence interval around the population variance, σ^2. If our critical values at a 90% confidence interval are 17.71 and 42.56, what is the range for σ^2?
10. What statement would you give surrounding this confidence interval?

Keywords

ANOVA test

Chi-square distribution

Chi-square statistic

Contingency table

Degrees of freedom

Goodness-of-fit test

Test for one variance

Test of homogeneity

Test of independence